Role of non-coplanarity in nuclear reactions using the Wong formula based on the proximity potential

Manie Bansal and Raj K. Gupta
Physics Department, Panjab University,Chandigarh-160014, INDIA

Introduction

Recently [1], we assessed Wong's formula [2] for its angular momentum ℓ-summation and "barrier modification" effects at sub-barrier energies in the dominant fusion-evaporation and capture (equivalently, quasi-fission) reaction cross-sections. For use of the multipole deformations (up to β_4) and (in-plane, Φ=0^0) orientations-dependent proximity potential in fusion-evaporation cross-sections of ^{58}Ni+^{58}Ni, ^{64}Ni+^{64}Ni and ^{100}Mo, known for fusion hindrance phenomenon in coupled-channels calculations, and the capture cross-sections of ^{48}Ca+^{238}U, ^{244}Pu and ^{248}Cm reactions, forming superheavy nuclei, though the simple ℓ=0 barrier-based Wong formula is found inadequate, its extended version, the ℓ-summed Wong expression fits very well the above noted capture cross-sections at all center-of-mass energies $E_{c.m.}$'s, but require (additional) modifications of the barriers to fit the fusion-evaporation cross-sections in the Ni-based reactions at below-barrier energies. Some barrier modification effects are shown [1] to be already present in Wong expression due to its inbuilt ℓ-dependence via ℓ-summation.

In this paper, we study for the first time the dynamics of fission reactions, such as ^{11}B+^{235}U and ^{14}N+^{232}Th forming ^{246}Bk* [3], on the basis of the extended, ℓ-summed Wong formula, including also the non-coplanarity ($\Phi \neq 0^0$) degree-of-freedom for all the three types of reactions, the fusion-evaporation, capture and fission cross-sections.

The extended Wong model

Wong's expression for fusion cross-section due to colliding two deformed and oriented nuclei (orientations θ_i), lying in two different planes (azimuthal angle Φ between the planes), in terms of ℓ partial waves, is

$$\sigma(E_{c.m.},\theta_i,\Phi) = \frac{\pi}{k^2}\sum_{\ell=0}^{\ell_{max}}(2\ell+1)P_\ell(E_{c.m.},\theta_i,\Phi), \quad (1)$$

with $k=\sqrt{\frac{2\mu E_{c.m.}}{\hbar^2}}$, and μ, the reduced mass. P_ℓ is the transmission coefficient for each ℓ, describing, in Hill-Wheeler approximation, the penetration of barrier $V_\ell(R,E_{c.m.},\theta_i,\Phi)$.

Instead of solving Eq. (1) explicitly, which require the complete ℓ-dependent potentials $V_\ell(R,E_{c.m.},\theta_i,\Phi)$, Wong summed it up *approximately*, using only ℓ=0 quantities, which on replacing the ℓ-summation in (1) by an integral, gives the Wong formula [2]

$$\begin{aligned}&\sigma(E_{c.m.},\theta_i,\Phi)\\ &= \frac{{R_B^0}^2\hbar\omega_0}{2E_{c.m.}}\ln\left[1+\exp\left(\frac{2\pi}{\hbar\omega_0}(E_{c.m.}-V_B^0)\right)\right].\end{aligned} \quad (2)$$

Integrating (2) over θ_i and Φ, we get the fusion cross-section $\sigma(E_{c.m.})$.

For an explicit summation over ℓ in Eq. (1), the ℓ-dependent interaction potential $V_\ell(R)$ is a sum of Coloumb and nuclear proximity and centrifugal potentials, as

$$\begin{aligned}V_\ell(R) &= V_P(R,A_i,\beta_{\lambda i},T,\theta_i,\Phi)+\frac{\hbar^2\ell(\ell+1)}{2\mu R^2}\\ &\quad +V_C(R,Z_i,\beta_{\lambda i},T,\theta_i,\Phi),\end{aligned} \quad (3)$$

where, the ℓ-summation in Eq. (1) is then carried out for the ℓ_{max} determined empirically for a best fit to measured cross-section. This procedure of explicit ℓ-summation works very well for Φ=0^0 case [1] in capture reactions ^{48}Ca+^{238}U, ^{244}Pu and ^{248}Cm, but require further modification of the barrier for Ni-based reactions at sub-barrier energies, which could be carry out empirically [1] by either (i) keeping the curvature $\hbar\omega_\ell$ same and modifying the

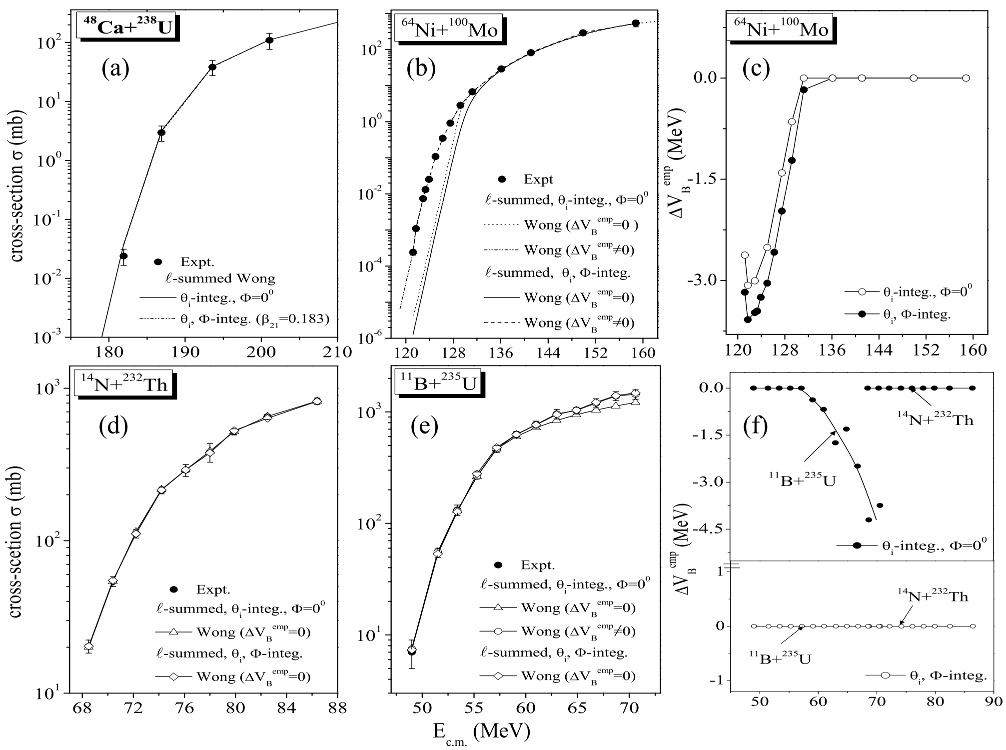

FIG. 1: ℓ-summed Wong results for both Φ=0^0 and $\Phi \neq 0^0$, compared with experimental data, for: (a) ^{48}Ca+^{238}U (b) and (c) ^{64}Ni+^{100}Mo, and (d) to (f) ^{246}Bk* due to ^{14}N+^{232}Th and ^{11}B+^{235}U channels.

barrier height V_B^ℓ, as

$$V_B^\ell(modified) = V_B^\ell + \Delta V_B^{emp},$$

or (ii) keep the barrier height V_B^ℓ same and modify the curvature $\hbar\omega_\ell$. We use here the method of modifying the barrier height.

Calculations and results

The results of ℓ-summed Wong expression (1) for both the cases of Φ=0^0 and $\Phi \neq 0^0$ are given in Fig. 1 for all the three types of reactions. Fig. 1(a) shows that the capture cross-section in ^{48}Ca+^{238}U is fitted nicely even after giving a small deformation (β_{21}=0.183) to ^{48}Ca for carrying out Φ-integration. The fitted $\ell_{max}(E_{c.m.})$ increase by one-to-two units. Similarly, the ^{64}Ni+^{100}Mo reaction is fitted for both Φ=0^0 and $\Phi \neq 0^0$ by allowing empirically a small increase in "barrier lowering" ΔV_B^{emp} (Figs. 1(b) and 1(c)). On the other hand, there is a strong entrance channel dependence in the case of fission reaction: whereas a nice fit is obtained for both Φ=0^0 and $\Phi \neq 0^0$ cases in ^{14}N+^{232}Th channel (Fig. 1(d)), a large disagreement in cross-sections at higher energies (Fig. 1(e)) and hence a large "barrier lowering" ΔV_B^{emp} (Fig. 1(f)) is obtained for Φ=0^0 in ^{11}B+^{235}U channel, which reduces to zero for $\Phi \neq 0^0$ case. In other words, for fission of ^{246}Bk*, the inclusion of non-coplanarity gives a complete fit to data for both the reaction channels, without introducing ΔV_B^{emp}.

References

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